Eliminate the parameter to find a Cartesian equation for the curve with parametric equations $\begin{array}{l} x = 2\sinh t \\ y = 3\cosh t \end{array}$. **SCORE: _____ / 4 PTS**

Also, based on the Cartesian equation, name the shape of the curve (the answer should be only one word).

$\cosh^2 t - \sinh^2 t = 1$

$\frac{1}{2}$ 1 $\frac{1}{2}$

$\frac{y^2}{9} - \frac{x^2}{4} = 1$, 1

HYPERBOLA, 1

An object moves in a straight line from the point $(5, -9)$ at time $t = 0$, to the point $(-1, -2)$ at time $t = 1$. **SCORE: _____ / 4 PTS**

Find parametric equations for the object's motion.

$x = 5 + (-1-5)t$

$y = -9 + (-2 - {}^-9)t$

$\longrightarrow$

1 1 2

$x = 5 - 6t$

$y = -9 + 7t$

An object moves clockwise along the circle $x^2 + (y-1)^2 = 4$, starting at the point $(0, -1)$ at time $t = 0$, and ending at the same point at time $t = 2\pi$. Find parametric equations for the object's motion. **SCORE: _____ / 6 PTS**

CENTER $(0, 1)$

RADIUS 2

$\longrightarrow$

$x = 0 + 2\cos t$

$y = 1 + 2\sin t$

$t = \pi$

$s = \frac{\pi}{2}$

$t = 0$

$s = 0$

$t = \frac{3\pi}{2}$

$s + t = \frac{3\pi}{2}$

$t = \frac{3\pi}{2} - s$

$x = 2\cos\left(\frac{3\pi}{2} - s\right)$

$y = 1 + 2\sin\left(\frac{3\pi}{2} - s\right)$

$\longrightarrow$

1 1 1

$x = -2\sin s$

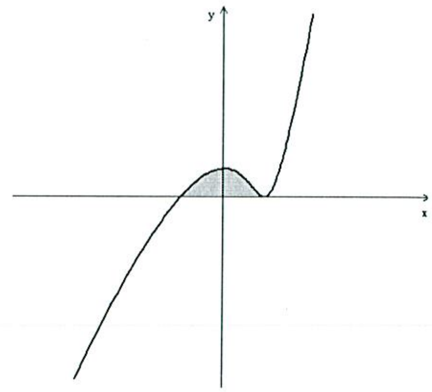
$y = 1 - 2\cos s$

1 1 1

The graph on the right corresponds to the parametric equations $\begin{aligned} x &= 2(1-t-t^3) \\ y &= t^2-t^3 \end{aligned}$.

SCORE: _____ / 16 PTS

NOTE: The graph has been distorted: the $x-$ and $y-$axes use different scales.



[a] Find the $x-$ and $y-$coordinates of all $x-$intercepts **algebraically**.
NOTE: You are NOT allowed to just plug in points , nor use guess and check.

$y=0 \longrightarrow t^2-t^3=0$

$t^2(1-t)=0$

$t=0, 1$

① POINT EACH

@ $t=0$ $(x,y)=(2,0)$

@ $t=1$ $(x,y)=(-2,0)$

[b] Write, **BUT DO NOT EVALUATE**, a dt integral for the shaded area.

MOVING FROM RIGHT @ $t=0$ TO LEFT @ $t=1$

$$A = -\int_0^1 (t^2-t^3)(2(-1-3t^2))\,dt$$

← SUBTRACT $\frac{1}{2}$ IF YOU DIDN'T WRITE dt

[c] Find the equation of the tangent line at the **left** $x-$intercept
(ie. at the $x-$intercept where $x<0$).

$$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{2t-3t^2}{-2(1+3t^2)}\right|_{t=1} = \frac{-1}{-8} = \frac{1}{8}$$

$$y-0=\frac{1}{8}(x-{}^-2)$$

SUBTRACT $\frac{1}{2}$ IF YOU DIDN'T WRITE "$y=$" $\rightarrow$ $y=\frac{1}{8}(x+2)$

[d] Find $\dfrac{d^2y}{dx^2}$ at the **right** $x-$intercept.

$$\left.\frac{d^2y}{dx^2}\right|_{t=0} = \left.\frac{\frac{d}{dt}\frac{2t-3t^2}{-2(1+3t^2)}}{-2(1+3t^2)}\right|_{t=0} = \left.\frac{(2-6t)(-2(1+3t^2))-(2t-3t^2)(-2(6t))}{(-2(1+3t^2))^3}\right|_{t=0}$$

$$=\frac{2(-2)(1)}{(-2(1))^3}$$

$$=\frac{-4}{-8}=\frac{1}{2}$$